Adama Science and Technology University
School of Applied Natural Science
Department of Applied Mathematics
Applied Mathematics III Math 2021
Assignment II

Submission date April 25, 2023
Maximum mark 10

Instruction: Show all the necessary steps clearly

1. Suppose the mass density at a given point on a thin wire is equal to the square of the distance from that point to the x axis. If the wire is helical and is parametrized by

$$r(t) = sin(t)i - cos(t)j + 4tk \quad \text{for } \pi \leq t \leq 2\pi$$

find the mass m of the wire. (1 point)

2. Let $F(x, y, z) = (3x^2y + 2yz^2)i + (x^3 + 2xz^2 + 2y)j + (4xyz + 4)k$ and C be any curve from $(1, 2, -4)$ to $(5, 0, 2)$, then

   (a) Show that F is conservative. (0.5 point)

   (b) Find potential function. (1 point)

   (c) Evaluate $\int_C F(r).dr$. (0.5 point)

3. Let u be continuous with continuous first and second derivatives on a simple closed path C and throughout the interior D of C. Show that

$$\int_C -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy = \int\int_D \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) dA.$$

(2 points)

4. Find the center of mass of part of paraboloid $z = 16 - x^2 - y^2$ lying in the first octant between cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ with density function $\delta(x, y, z) = \frac{xy}{\sqrt{4x^2 + 4y^2 - 1}}$. (2 points)

5. Let Σ consist of the portion of the paraboloid

$$2z = x^2 + y^2$$

below the plane $z = x$, and orient Σ by the normal directed downward. Suppose a fluid is passing through Σ with a velocity v given by

$$v(x, y, z) = 2yzi + 2xzj + 5yk.$$

Find the circulation of the fluid around the boundary of Σ. (2 points)

6. Show that $u(x, y) = \frac{1}{2}(x^2 - y^2)$ is harmonic, that is, solution of Laplace equations and find harmonic conjugate $v(x, y)$ of $u(x, y)$. (1 point)